Roll No. : ....

Total No. of Questions : 9] [Total No. of Printed Pages : 6

BCA-1005

BCA (Semester-I) (NEP) Examination, 2024-25

(Major)

MATHEMATICS-I

Time : 2 Hours] [Maximum Marks : 75

Note : 1. Attempt questions from **all** sections as directed.

2. The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

3. "B" copy will not be provided.

Section-A

Short Answer Type Questions

Note : All questions are **compulsory**. Each question carries **5** marks. [9×5=45]

1. (a) Write the cofactors of each elements of the determinant $\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix}$.

(b) State Lagrange's mean value theorem and find the value of c for $f(x) = x(x-1)(x-2)$ in interval $\left[0, \frac{1}{2}\right]$.

(c) If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ prove that $(2I + 3E)^3 = 8I + 36E$.

(d) Evaluate $\lim\limits_{x \to 3} \frac{x^2 - 5^2}{x - 5}$.

(e) Find the angle between $\vec{a} = 2i + 2j - k$ and $\vec{b} = 3j + 2k$.

(f) If $y = (\log x)^3$, then find $\frac{dy}{dx}$.

(g) Evaluate indefinite integral $\int \frac{1}{\sqrt{9 + x^2}} dx$.

(h) If A, B are square matrices, such that $A^2 = A$ and $B^2 = B$ show that $(AB)^2 = AB$, if A, B commute.

(i) Find the volume of parallelepiped with adjacent vertices vectors

$\vec{a} = 3i + 2j - k, b = i + j - k, c = 2i - k$

Section-B

Long Answer Type Questions

Note : Attempt **any one** question from the following. Each question carries **15** marks. [15×1=15]

2. (a) Find the differential coefficient of $\frac{e^x}{e^x + 1}$.

(b) Evaluate $\lim\limits_{x \to 0} \frac{x^2 \sin \frac{1}{x}}{e^x}$.

(c) Discuss the continuity of the greatest integer function $f(x)=[x]$ at $x=\frac{1}{2}$.

3. (a) For what values of x and y the matrices A and B are equal :

$$A=\begin{bmatrix}2x+1 & 3y \\ 0 & y^2-5y\end{bmatrix} \text{ and } B=\begin{bmatrix}x+3 & y^2+2 \\ 0 & -6\end{bmatrix}$$

(b) Find Maclaurin's series expansion of function $e^{\sin x}$ about the point $x=0$.

(c) Prove that the maximum value of $f(x)=\sin x+\cos x$ is $\sqrt{2}$.

4. (a) Write statement of Cayley-Hamilton theorem and find characteristic equation of matrix

$$A=\begin{bmatrix}2 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 1\end{bmatrix}.$$

(b) Verify Cayley - Hamilton theorem for the matrix $A=\begin{bmatrix}2 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1\end{bmatrix}$.

(c) Solve the equation $\begin{vmatrix}3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8\end{vmatrix}=0$.

5. (a) Find the derivative of $\frac{1}{(x+a)(x+b)(x+c)}$.

(b) Find the inverse of the matrix $A=\begin{bmatrix}2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & 2\end{bmatrix}$ using adjoint method.

(c) Prove that $\lim_{x\to 0}(1+px)^{1/x}=e^P$.

Section-C

Long Answer Type Questions

Note : Attempt **any one** question from the following. Each question carries **15** marks. [15×1=15]

6. (a) If $f(x)=x^3+8x^2+15x-24$, calculate the value of $f\left(\frac{11}{10}\right)$ by the application of Taylor's series.

(b) Evaluate indefinite integral

7. (a) Find the unit vector perpendicular to both the vectors $\vec{a} = 3\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$.

(b) Evaluate integral by parts $x \tan^{-1} x dx$

8. (a) If $\cos y = x \cos(b + y)$, find $\frac{dy}{dx}$.

(b) Evaluate integral $\int \frac{1}{x^6 + x^4} dx$.

9. (a) Find the work done by a force of magnitude 3 units acting in the direction $3i - 2j + 6k$ acting on a particle, which is displaced from the point $A(2, -3, -1)$ to point $B(4, 3, 1)$.

(b) Verify Rolle's Theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$.

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